

Thursday 3rd March, 2016, 01:21

Radiation from an emitter in the ghost free scalar theory

Valeri P. Frolov and Andrei Zelnikov

Theoretical Physics Institute, Department of Physics

University of Alberta, Edmonton, AB, Canada T6G 2E1

We study radiation emitted by a time-dependent source of a scalar massless field in the framework of the ghost-free modifications of the theory. We consider a simple model of the emitter: namely, we assume that it is point-like and monochromatic. We focused on the most common versions of the ghost-free theory, where the propagator \square^{-1} is modified as follows $\exp(-(\square/\mu^2)^N)\square^{-1}$, where μ is the characteristic mass-scale of such GF_N -theory. We demonstrated that far from the source, in the wave-zone, the radiation asymptotically converges to its "classical" value for any $N \geq 1$. However, in the near-zone the behavior of the field is quite different from the "classical" case. The difference of field amplitude for the ghost-free field and for the classical one has an oscillatory behavior in this domain. A number of oscillations increases with N . The amplitude of these oscillations remain finite for even N , while it infinitely grows with frequency for odd N . This behavior indicates that even in the classical domain GF_N theories might have pathological behavior.

PACS numbers: 04.70.-s, 04.50.Kd

I. INTRODUCTION

A well known problem of the General Relativity is its ultra-violet (UV) incompleteness. In the classical theory it manifests itself as existence of singularities (see e.g. [1]). In a quantum regime the General Relativity is non-renormalizable. It is expected that both of the problems can be "cured" by admitting higher in curvature terms the the gravitational action [2–6]. However, such higher-derivative theories inevitably has a common severe problem: they contain ghosts [7]. In a general case, this additional unphysical degrees of freedom with negative energy result in classical and quantum instability of the theory [8–10]. The

problem of ghosts can be solved if one allows an infinite number of derivatives in the action of the field, that makes it non-local. This can be achieved by a special modification of the standard propagator. Suppose one considers a scalar theory with the propagator $(\square - m^2)^{-1}$ and modifies it as follows $[a(\square)(\square - m^2)]^{-1}$, where $a(z)$ is an entire function of the complex variable z , which does not have poles in the complex plane of z . For such a modification the propagator has only one pole, which describes a particle with mass m . Such ghost-free theories were widely discussed mainly in connection of the well known problem of singularities in gravity (see, e.g. [11–17]). Their application to the problems of cosmology [18–21] and black holes can be found in [22–28].

The simplest choice of the form-factor $a(z)$, which is usually adopted in the publications, is

$$a(z) = \exp[(z/\mu^2)^N]. \quad (1.1)$$

We shall use the notation GF_N for such a theory. The mass μ determines the energy scale where ghost-free modification of the theory becomes important. In the limit $\mu \rightarrow \infty$ one has $a(0) = 1$, so that the theory reproduces correctly the results of the standard theory.

There are several papers discussing the gravitational field of a point-mass in the linearized version of the ghost-free gravity [29–34]. The main conclusion is that the gravitational potential in the ghost-free modified theory is finite and regular at the location of the point mass, while at far distance it converges to the standard Newtonian potential. This result is valid for any value of N . At the same time one can expect that theories with odd value of N may have "bad" behavior for time-dependent sources. The reason for this is the following. In a static case \square reduces to the Laplace operator Δ . The operator $-\Delta$ is non-negative definite as well as any power of it. In the general time-dependent case, the \square operator has both positive and negative eigen-values, which for odd N may lead to instability. The purpose of this paper is to analyze behavior of radiation emitted by a time-dependent source in the ghost-free theory with different values of N .

II. SCALAR FIELD FROM A MONOPOLE EMITTER

We restrict ourselves by considering a ghost-free scalar field in four dimensional spacetime and use a simple model of a monopole point-like emitter. These restrictions play mainly technical role and the obtained results can be generalized to the case of the ghost-free fields

with non-zero spin in higher dimensions. Similarly, the model of the emitter can be modified. Our starting point is the action

$$S = \int dx |g|^{1/2} \left(\frac{1}{8\pi} \varphi a(\square) (\square - m^2) \varphi + j \varphi \right). \quad (2.1)$$

where the entire function $a(z)$ is given by (1.1). The corresponding field equation reads

$$a(\square) (\square - m^2) \varphi = -4\pi j. \quad (2.2)$$

For simplicity we consider a massless case $m = 0$. We assume that the source of the field φ is point-like and its scalar charge oscillates with the frequency Ω . The equation (2.2) is linear and its coefficients are real. It is convenient to choose its source j in the form $j = \text{Re } \hat{j}$, where

$$\hat{j}(x) = q e^{i\Omega t} \delta(\mathbf{x}), \quad x = (t, \mathbf{x}). \quad (2.3)$$

After performing the calculations and getting the complex field $\hat{\varphi}$ created by this complex source (2.3) one can obtain the value of the field φ by taking the real part of this complex answer. The complex scalar field $\hat{\varphi}$ created by the charge can be expressed in terms of the retarded Green function $G_{\text{Ret}}(x, x')$

$$\hat{\varphi}(x) = 4\pi \int dx' G_{\text{Ret}}(x, x') \hat{j}(x'). \quad (2.4)$$

In momentum representation one has

$$\hat{\varphi}(t, \mathbf{x}) = \frac{q}{2\pi^2} \int d\omega d\mathbf{p} e^{-i\omega t + i\mathbf{p}\mathbf{x}} G_{\text{Ret}}(\omega, \mathbf{p}) \delta(\omega + \Omega), \quad (2.5)$$

$$G_{\text{Ret}}(x, x') = \int \frac{d\omega d\mathbf{p}}{(2\pi)^4} e^{-i\omega(t-t') + i\mathbf{p}(\mathbf{x}-\mathbf{x}')} G_{\text{Ret}}(\omega, \mathbf{p}). \quad (2.6)$$

In the case of GF_N theory the retarded Green function can be written as

$$G_{\text{Ret}}(\omega, \mathbf{p}) = G_{\text{Ret}}(\omega, p) = \frac{\exp \left[- \left(\frac{p^2 - \omega^2}{\mu^2} \right)^N \right]}{(\omega + i\epsilon)^2 - p^2}, \quad p = \sqrt{\mathbf{p}^2}, \quad (2.7)$$

$$= \mathcal{P} \frac{1}{\omega^2 - p^2} e^{-\left(\frac{p^2 - \omega^2}{\mu^2} \right)^N} - i\pi \text{sgn}(\omega) \delta(\omega^2 - p^2). \quad (2.8)$$

Then, after integration over angles, we get ($r = \sqrt{\mathbf{x}^2}$)

$$\hat{\varphi}(t, \mathbf{x}) = \frac{2q}{\pi r} \int_0^\infty dp p \sin(pr) e^{i\Omega t} G_{\text{Ret}}(-\Omega, p) \quad (2.9)$$

$$= \frac{q e^{i\Omega t}}{r} \left[\frac{2}{\pi} \int_0^\infty dp \frac{p \sin(pr)}{\Omega^2 - p^2} e^{-\left(\frac{p^2 - \Omega^2}{\mu^2} \right)^N} + i \sin(\Omega r) \right] \quad (2.10)$$

$$= \frac{q e^{i\Omega t}}{r} \left[\frac{1}{\pi} \int_{-\infty}^\infty dp \frac{\sin(pr)}{\Omega - p} e^{-\left(\frac{p^2 - \Omega^2}{\mu^2} \right)^N} + i \sin(\Omega r) \right]. \quad (2.11)$$

To obtain the last relation we used the fact that the integrand in (2.10) is an even function of p . In the last two lines we used a notation \oint to indicate that the corresponding integral should be calculated according to the principal value prescription. In the limit $\mu = \infty$ the GF_N theories reduce to the ordinary massless scalar theory, satisfying the field equation

$$\square \hat{\varphi}_0(t, \mathbf{x}) = -4\pi \hat{j}. \quad (2.12)$$

In this case the principal value integral (see (2.11)) has exactly the form of the Hilbert transform of $\sin(pr)$ (see [35] table 1.5 Eq.(5.1))

$$\frac{1}{\pi} \oint_{-\infty}^{\infty} dp \frac{\sin(pr)}{\Omega - p} = -\cos(\Omega r). \quad (2.13)$$

Thus, the result of the integration becomes

$$\hat{\varphi}_0(t, \mathbf{x}) = \frac{q e^{i\Omega t}}{r} \left[\frac{1}{\pi} \oint_{-\infty}^{\infty} dp \frac{\sin(pr)}{\Omega - p} + i \sin(\Omega r) \right] = -\frac{q e^{i\Omega(t-r)}}{r}. \quad (2.14)$$

Let us consider the difference of the solution for the scalar field $\hat{\varphi}(t, \mathbf{x})$ created by the source (2.3) in GF_N theory and in the ordinary massless scalar theory

$$\hat{\psi}(t, \mathbf{x}) = \hat{\varphi}(t, \mathbf{x}) - \hat{\varphi}_0(t, \mathbf{x}), \quad (2.15)$$

$$\hat{\psi}(t, \mathbf{x}) = \frac{q e^{i\Omega t}}{r} h(\Omega, r), \quad (2.16)$$

where

$$h(\Omega, r) = \frac{2}{\pi} \int_0^{\infty} dp \sin(pr) f(p), \quad f(p) = \frac{p}{\Omega^2 - p^2} \left[e^{-\left(\frac{p^2 - \Omega^2}{\mu^2}\right)^N} - 1 \right]. \quad (2.17)$$

For all $N \geq 1$ the function $f(p)$ is finite at $p = \Omega$ and, hence, the principal value integral reduces to the ordinary one. $f(p) \in C^\infty$ is a smooth function on the interval $[-\infty, \infty]$ decreasing at infinity as p^{-1} and satisfying the condition $f(-p) = -f(p)$. Using these properties one can estimate an asymptotic of the function $h(\Omega, r)$ at large r . Note that this function has the form of the Fourier transform of $f(p)$

$$h(\Omega, r) = \frac{2}{\pi} \text{Im} \int_0^{\infty} dp \exp(ipr) f(p). \quad (2.18)$$

One can show (see, e.g., [36] page 91, Eq.(1.10)) that at large distances $r \gg \mu^{-1}$

$$h(\Omega, r) = \sum_{l=0}^n (-1)^{l+1} r^{-2l-1} \frac{d^{2l}}{dp^{2l}} f(p) \Big|_{p=0} + O(r^{-2n-1}). \quad (2.19)$$

Because of the symmetry property $f(-p) = -f(p)$ all even derivatives of $\partial_p^{2l} f(p)$ vanish at $p = 0$. As the result of (2.19) and this property, $h(\Omega, r)$ falls off at large r faster than any power of $1/r$. This means that for all GF_N theories the nonlocal effects become important only at distances from the charge of the order of μ^{-1} . At large distances $r \gg \mu^{-1}$ only the usual asymptotic

$$\hat{\varphi}(t, \mathbf{x}) \sim \hat{\varphi}_0(t, \mathbf{x}) = -\frac{q e^{i\Omega(t-r)}}{r}, \quad \varphi(t, \mathbf{x}) = \text{Re}(\hat{\varphi}(t, \mathbf{x})) \quad (2.20)$$

survives.

At $r = 0$ the scalar field φ is finite for all GF_N theories with $N \geq 1$. It has the form

$$\hat{\varphi}(t, 0) = q e^{i\Omega t} \eta(\Omega), \quad \eta(\Omega) = i\Omega + \frac{2}{\pi} \int_0^\infty dp \frac{p^2}{\Omega^2 - p^2} e^{-\left(\frac{p^2 - \Omega^2}{\mu^2}\right)^N}. \quad (2.21)$$

III. NONLOCAL EFFECTS IN GF_N SCALAR THEORY

Consider an example of GF_2 scalar theory in more detail. The ghost-free nonlocal contributions to the massless scalar theory are described by the field ψ . It describes standing waves with the amplitude at large radii falling off faster than any power of $1/r$. The field ψ (see (2.16)-(2.17)) is proportional to the dimensionless function $h(\Omega, r)$ which depends on the dimensionless combinations Ω/μ and μr only. For various values of Ω the dependence of $h(\Omega, r)$ on the radius is depicted in Figs.1-2. The potential created by a point source (2.3) in the ghost-free models is

$$\hat{\varphi}(t, \mathbf{x}) = -\frac{q e^{i\Omega t}}{r} [e^{-i\Omega r} - h(\Omega, r)]. \quad (3.1)$$

Notice that in ghost-free models at small r the function $h(\Omega, r) = 1 + O(r)$ and the potential is finite at $r = 0$ (see (2.21)). All GF_N theories with even N look alike. For example, $h(\Omega, r)$ in the case of GF_4 theory is depicted in Fig.3

Now let us consider GF_N theories of the odd order N . They also look similar to each other. A typical example is GF_1 scalar theory. It reveals the key properties of the whole class of theories of the odd order. In this particular case one can compute the field of the oscillating monopole charge analytically. The function (2.17) takes the form of the Hilbert transform (see [35])

$$h(\Omega, r) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dp}{\Omega - p} \sin(pr) \left[e^{-\frac{p^2 - \Omega^2}{\mu^2}} - 1 \right] \quad (3.2)$$

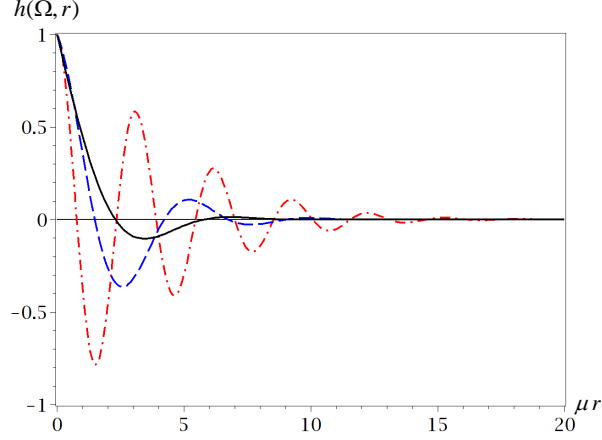


FIG. 1: $h(\Omega, r)$ in GF_2 theory as the function of r for $\Omega/\mu = 0$ (solid line), $\Omega/\mu = 1$ (dashed line), and $\Omega/\mu = 2$ (dash-dotted line).

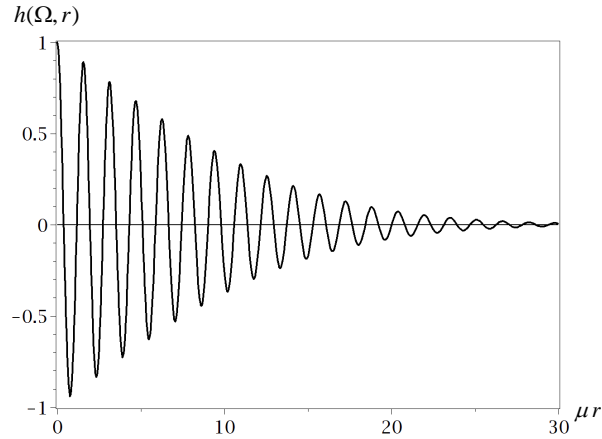


FIG. 2: $h(\Omega, r)$ in GF_2 theory as the function of r for $\Omega/\mu = 4$.

and can be computed explicitly (see [35] table 1.5 Eqs.(5.1),(5.138))

$$h(\Omega, r) = \cos(\Omega r) - \text{Re} \left[e^{i\Omega r} \text{erf} \left(\frac{\mu r}{2} + i \frac{\Omega}{\mu} \right) \right]. \quad (3.3)$$

For small frequencies $\Omega \ll \mu$ this function behaves similarly to that in the even N theories. The crucial difference appears at $\Omega > \mu$ (see Fig.4). Comparing this plot with Figs.1-2 one can see that, while in all even N amplitude the nonlocal correction $h(\Omega, r)$ is always within the interval $[-1, 1]$, in the GF_1 theory near $r \sim \mu^{-1}$ it exponentially grows with growing Ω . This behavior can be derived from the asymptotic of the error function erf of a complex argument. At $\Omega \gg \mu^2 r$ we can use the asymptotic expansion of the erf function for a

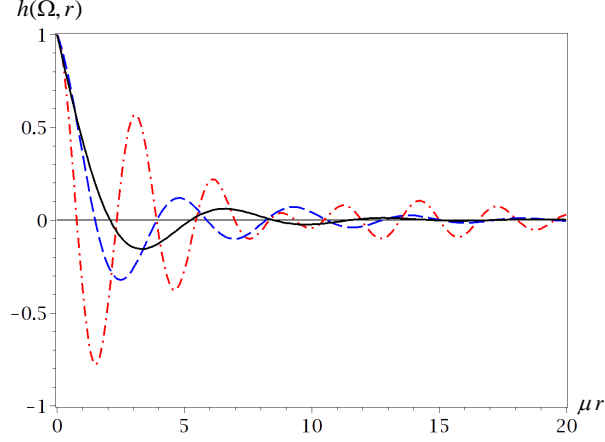


FIG. 3: $h(\Omega, r)$ in GF_4 theory as the function of r for $\Omega/\mu = 0$ (solid line), $\Omega/\mu = 1$ (dashed line), and $\Omega/\mu = 2$ (dash-dotted line).

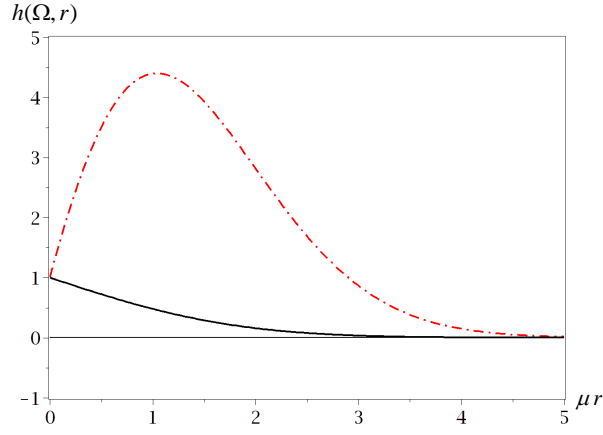


FIG. 4: $h(\Omega, r)$ in GF_1 theory as the function of r for $\Omega/\mu = 0$ (solid line) and $\Omega/\mu = 2$ (dash-dotted line)

complex argument z (see, e.g., [37] Eq. (7.12.1))

$$\operatorname{erf}(z) = 1 + \frac{1}{\sqrt{\pi}z} e^{-z^2} \left(1 + \frac{1}{2z^2} + \dots\right), \quad |z| \rightarrow \infty, \quad |\arg z| < \frac{3\pi}{4}. \quad (3.4)$$

Then we obtain

$$h(\Omega, r) = \frac{\mu^3 r}{2\sqrt{\pi}\Omega^2} e^{\Omega^2/\mu^2} (1 + O(\Omega^{-2})). \quad (3.5)$$

Thus, at any finite $r \sim \mu^{-1}$ and large frequencies, the nonlocal corrections to the ordinary scalar field become exponentially large $\sim \exp(\Omega^2/\mu^2)$. This is a reflection of a strong instability for the field of time-dependent sources in GF_1 theory. This instability is an inherent property of all GF_N with odd N .

IV. DISCUSSION

Ghost-free models have been intensively discussed recently. The ghost-free modification of the gravity improves its UV behavior both in the classical and quantum domains. This opens an interesting possibility that the long-standing problems of the General relativity, its black-hole and cosmological singularities, can be resolved in the framework of the properly chosen ghost-free model. In this paper we study some properties of the ghost-free theories with time-dependent sources of the field. We made a number of simplifying assumptions. Instead of the gravity, we consider a scalar massless field, and we choose a simple model of the time-dependent source: a point-like emitter of the monochromatic radiation. We examined so called GF_N models of the ghost-free theory. We obtained solutions for the scalar field in such a model and demonstrated that at far distance, in the wave zone, this field asymptotically converges to its "classical" value. This happens for any $N \geq 1$. However, the structure of the field in the near zone is qualitatively different for odd and even values of N . Namely, the field remains bounded for even N , while for odd N its amplitude infinitely grows with the frequency of the emitter. This result supports a conclusion that all GF_N theories with odd N have a potential problem: their solutions may be unstable. This means that one can expect similar pathology of such theories in a more general set-up.

Acknowledgments

The authors thank the Natural Sciences and Engineering Research Council of Canada and the Killam Trust for their financial support. A part of this work was done during V.F.'s stay at the University of Oldenburg. He gratefully acknowledges support by the DFG Research Training Group 1620 Models of Gravity and thanks Prof. Jutta Kunz for her kind hospitality.

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